Core Principles

CCSD CLARK COUNTY SCHOOL DISTRICT

Geometry **8.G**

Understand and apply the Pythagorean Theorem.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

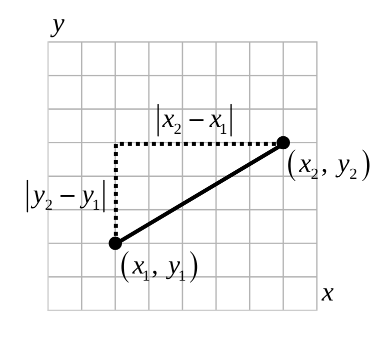
A fundamental skill when working in any coordinate system is the ability to find the distance between two points. The coordinate system need not be Cartesian, but that will be the focus here.

Instruction on this standard should be done incrementally, beginning with what students already know and progressing from more concrete representations to more abstract ones. What should *not* be done is simply hand students the formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and have them start plugging in points. Students have already learned numbers can be represented as a distance from 0 on a number line. In the coordinate plane, this would be the distance from the origin to another point on either axis. Students also know the distance between two points in a plane with the same first or second coordinate is the absolute value of the other coordinates' difference, like the distance between (2, 3) and (6, 3) is 4. Building on these ideas and the Pythagorean Theorem, have students then look at the distance between the origin and a point not on an axis, then progressing to two points off the origin where the horizontal and vertical coordinates are both different, like (2, 3) and (5, 4). Coordinates should start with whole numbers, then extend to integers, then to signed rational numbers.

Through all of this, students do not *need* the distance formula. Once students have mastered the skill of computing the distance between two points, move to the abstract and *develop* the formula. Students should be guided to see that the distance between two arbitrary points (x_1, y_1) and (x_2, y_2) can be expressed as $d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$. Students should also reason that it does not matter which is point number 1 and which is point number 2.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



If that is not the distance formula you are familiar with, think about what students know: the horizontal and vertical distances—the legs of the right triangle—are absolute values of the differences. That formula is absolutely correct. Yet, this would be a good time to engage in a discussion of whether finding the absolute value is even necessary, considering we are going to square it anyway. Students should see in the structure of the formula that finding the absolute value is an unnecessary step. Writing the formula in the more traditional form also sets the stage for equations of circles in high school geometry. The concept of distance in a coordinate system will also be useful in future courses covering vectors and complex numbers.

Since a companion standard in grade 8 is using the Pythagorean Theorem in *three* dimensions, it wouldn't hurt to think about distances in space—points have coordinates of *x*, *y*, and *z*. Enterprising students may even be able to derive a formula, but all should be able to find the distance without one.

When assessing this standard, students should be able to work with distances in several ways: given two points, find the distance; given one point and the distance, find another point; and given a distance, find two points. Assessments should also include applications—geography, navigation, and video games are obvious choices.

Connections

5.G.1–2
6.NS.5–8
6.SP.5
7.NS.1
7.G.1
7.SP.3
8.NS.1–2
8.G.6–7
N.CN.4–6
N.VM.1–5
G.SRT.8–11
G.GPE.1
S.ID.6

Finally, this standard connects strongly to standard 8.NS.2, which requires students to *approximate* irrational numbers and locate them on a number line. Now students can locate them *exactly*. For example, to locate $\sqrt{13}$ on the number line, first find a point that lies $\sqrt{13}$ from the origin, like (2, 3). Using a compass, measure the distance from the origin to (2, 3), then mark a point on the *x*-axis (or *y*-axis) an equal distance from the origin. The point on the axis will be exactly at $\sqrt{13}$. This reinforces the connections among the Pythagorean Theorem, the distance formula, and properties of circles. Other values to consider are $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{10}$. For a challenge, have students try numbers like $\sqrt{3}$.