

Expressions and Equations

8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Students begin their exploration of *power* under the Common Core in Grade 6 by writing and evaluating expressions with exponents. Now, in Grade 8, they extend their knowledge of power to properties of exponents. Specifically, students should explore, learn, and apply eight properties of integer exponents in the real numbers.

Product of powers: $a^m \cdot a^n = a^{m+n}$

Negative powers: $a^{-m} = \frac{1}{a^m}$

Quotient of powers: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power: $(a^m)^n = a^{m \cdot n}$

First power: $a^1 = a$.

Power of a product: $(ab)^m = a^m \cdot b^m$

Zerth power: $a^0 = 1$, for all $a \neq 0$.

Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Mathematical Practices

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

That seems like a daunting list, but by building on patterns learned in Grade 6, students can quickly learn and apply the rules.

The first things not to do during instruction are to tell students the rules and begin to apply them. These are abstract concepts, particularly the zeroth and negative powers. Start with an extensive exploration of the product of powers. It shouldn't take long to discover when multiplying powers of the same base, one adds the exponents. Ultimately, we want them to understand the general rule in the abstract, but don't push too hard through the concrete phase.

The quotient of powers should be developed in a similar fashion using the notion of dividing out common factors in the numerator and denominator. Students should discover when dividing powers of the same base, one subtracts the exponents. From this, the first and zeroth power properties are discovered. The negative powers property can also be constructed from simplifying fractions. For example, $\frac{3^2}{3^4} = \frac{3 \times 3}{3 \times 3 \times 3 \times 3} = \frac{\cancel{3} \times \cancel{3}}{3 \times 3 \times \cancel{3} \times \cancel{3}} = \frac{1}{3^2}$. Using the quotient rule, $\frac{3^2}{3^4} = 3^{-2}$, so $3^{-2} = \frac{1}{3^2}$. Using similar numerical techniques, the remaining three properties are easily developed.

Students will probably enter Grade 8 knowing that any number to the first power is the number itself. But the "why" may elude them; it's not obvious. Using the quotient of powers property, it's quick to see why $3^1 = 3$, and why we don't have to write an exponent of 1.

Should a student be able to quote the general rules? Of course. But, the student who knows that $3^2 \times 3^4 = 3^6$ and can explain why, is preferable to one who can merely recite a rule from memory. Get the concept down first, then move to the abstract.

This standard has multiple connections in Grade 8. Also in *Expressions and Equations*, students work with radicals, solve simple quadratic and cubic equations of the form $x^2 = p$ and $x^3 = p$, and do operations with numbers in scientific notation. In *Geometry*, the volume of some solids can be defined as the product of base area and height. Since the base is measured in units² and the height is measured in units, the volume is measured in units² \times units = units³.

It's also important that students explore what the properties of powers *don't* tell us. Among common misconceptions that students self-generate are $3^2 + 2^2 = 5^2$ (adding bases), $3^2 + 3^2 = 3^4$ (adding exponents), $3^2 + 3^2 = 6^4$ (adding both), and $(3 + 2)^2 = 3^2 + 2^2$ (distributing an exponent). These misconceptions and others should be explored and students should explain why they are not true. As an extension, students should argue whether a rule can even be established for these situations.

Assessment should include an understanding of properties both forward and backward. That is, a student should be able to take the expression $(3^4)^2$ and write the equivalent 3^8 , as well as re-express 3^8 as $(3^2)^4$ or $(3^4)^2$. They should also be able to explain their thinking and communicate how the properties are derived.

Once students have mastered the properties of integer powers, they are set up for the next step in Algebra I, rational powers. Since $3^1 \times 3^1 = 3^2$, it follows that $3^{1/2} \times 3^{1/2} = 3^1$. It's a short step to recognizing that $3^{1/2}$ must equal $\sqrt{3}$. These principles are applied further to seeing structure in expressions, creating exponential functions, and learning about logarithms.

Connections

- 4.NF.1
- 5.NF.4–6
- 5.MD.3–5
- 6.EE.1–2
- 6.G.1–2
- 7.NS.1–2
- 7.EE.2
- 7.G.4, 6
- 8.EE.2–4
- 8.G.7–9
- N.RN.1
- A.SSE.1–3
- F.IF.7–8
- F.LE.4